Physics Problem Solving: Conservation Laws

For classical systems of bodies interacting (those that are not quantum), there are four main conservation laws:

- Law of momentum conservation
- Law of energy conservation
- Law of angular momentum conservation
- Law of electric charge conservation

Here we will focus on the first two only, those that relate to momentum and energy.

In solving physics problems relating to physical systems where bodies interact, we can use kinematic (using the SUVAT equations) and dynamical (using Newton's Laws) methods. Additionally we may be able to use the conservation laws which in some cases may be easier and faster to do.

In applying the conservation laws, three stages must be distinguished in the interaction:

-state of objects prior to interaction
-interaction itself
-state of objects after the interaction

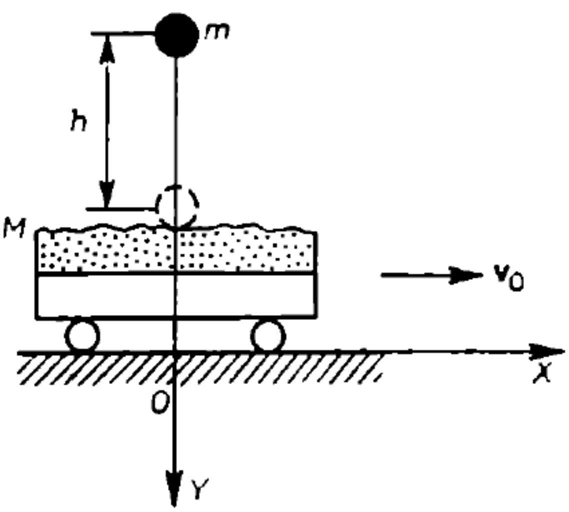
The following are the steps to solve the problem:

(i) Establish what objects are in the system
(ii) Check that the conditions for conservation are met
(iii) Select the inertial reference frame (coordinate system)
(iv) Find the value of the physical quantity prior to the interaction: A_1
(v) Determine the value of the physical quantity after the interaction: A_2
(vi) Write the conservation law:

$A_2 - A_1 = \Delta A = 0$

Example 1

A cart with sand whose combined mass M is 100 kg is moving in a straight line and uniformly along a horizontal surface with a speed $v_0 = 3\ ms^{-1}$. A ball of mass m = 20 kg falls onto the cart from a height h= 10m reckoned from the surface of the sand (initial speed of the ball is zero). Determine the speed of the cart-sand-ball system after the interaction. Friction can be neglected.



M = 100 kg

m = 20 kg

$V_0 = 3\ ms^{-1}$

h = 10 m

Solution 1

(i) The physical system consists of the cart with sand (single object) and the ball.

(ii) The system is not closed as there is an external force (gravity) acting on the ball. Hence, in general then, the conservation of momentum cannot be applied. However, in the direction of the movement of the cart, there is no external force and the component of gravity in this direction is zero. Hence in this direction, the conservation of momentum can be applied.

(iii) We select as the inertial reference frame with respect to the earth: the x-axis along the horizontal and the y-axis as the vertical.

(iv) The initial momentum along the x-axis is:

$$P_{1x} = Mv_0$$

(v) The final momentum along the x-axis is:

$P_{2x} = (M + m)\ v$
(where v is the final velocity of the cart-sand-ball object)

(vi) Writing the conservation law:

$P_{1x} = P_{2x}$

$Mv_0 = (M + m)\ v$

$$v = \frac{Mv_0}{(M+m)}$$

After substituting numerical values, $v = 2.5\ ms^{-1}$

<u>NOTE:</u>
We note that the answer is independent of h and so is not needed as it does not affect the final solution.

However, if we include the earth itself as a body part of a closed system, we may be able to calculate the speed the Earth gains as a result of the interaction with the ball. We could use the conservation of momentum and also the kinematics equations to find the speed of the Earth. The detailed explanation is slightly beyond the level of discussion here but you might get some idea with:

$M_{earth}v_{earth} = m_{ball}\ v_{ball}$

$$v_{earth} = \frac{m_{ball}}{M_{earth}}\ v_{ball}$$

$$= \frac{m_{ball}}{M_{earth}}\sqrt{2gh} \quad \text{(using } v^2 = u^2 + 2as\text{)}$$

Using $M_{earth} = 6.0 \times 10^{24\ kg}$, $g = 9.8\ ms^{-2}$

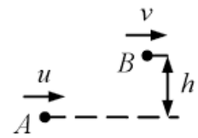
$V_{earth} \approx 5 \times 10^{-23}\ ms^{-1}$ (which is very very small)

Example 2

A particle of mass 0.50 kg moves from point A to point B under the action of it's weight and an external force. The particle's speed at A is 2.0 ms-1. Point A is at a height of 0.50 m below the level of B. As the particle moves from A to B, the work done by the external force on the particle is 3.0 J.

Find the speed of the particle at B.

m = 0.50 kg
u = 2 ms-1
h = 0.50 m
WD = 3.0 J



Solution 2

(i) The system consist of one particle with un-balanced forces acting on the particle – it's weight and an external force that moves it.

(ii) The total energy of the system is conserved if we consider the initial kinetic energy and work done and the final kinetic and potential energy. The external force does work and changes the gravitational and kinetic energy of the system.

(iii) We select the initial reference frame with respect to the earth and consider the horizontal and vertical motion of the particle.

(iv) The initial total energy is (where WD is the work done):

GPE + KE + WD = $mgh_0 + \frac{1}{2} m u^2$ + WD (where $h_0 = 0$ in our chosen coordinate system)

(v) After the interaction the final total energy is:

GPE + KE = $mg(h_0 + h) + \frac{1}{2} m v^2$

(vi) Writing out the conservation law:

$$mgh_0 + \frac{1}{2} m u^2 + WD = mg(h_0 + h) + \frac{1}{2} m v^2$$

$$\frac{1}{2} m v^2 = \frac{1}{2} m u^2 + WD - mgh$$

$$v = \sqrt{\frac{2}{m}\left(\frac{1}{2} m u^2 + WD - mgh\right)}$$

After substituting numerical values, $v = 2.5\ ms^{-1}$