TOPIC 1: MEASUREMENT

Measurement	**Learning Outcomes** Students should be able to:
Physical quantities and SI units	(a) recall the following base quantities and their SI units: mass (kg), length (m), time (s), current (A), temperature (K), amount of substance (mol). (b) express derived units as products or quotients of the base units and use the named units listed in 'Summary of Key Quantities, Symbols and Units' as appropriate. (c) use SI base units to check the homogeneity of physical equations. (d) show an understanding of and use the conventions for labelling graph axes and table columns as set out in the ASE publication *Signs, Symbols and Systematics (The ASE Companion to 16–19 Science, 2000)*. (e) use the following prefixes and their symbols to indicate decimal sub-multiples or multiples of both base and derived units: pico (p), nano (n), micro (μ), milli (m), centi (c), deci (d), kilo (k), mega (M), giga (G), tera (T). (f) make reasonable estimates of physical quantities included within the syllabus.
Scalars and vectors	(g) distinguish between scalar and vector quantities, and give examples of each. (h) add and subtract coplanar vectors. (i) represent a vector as two perpendicular components.
Errors and uncertainties	(j) show an understanding of the distinction between systematic errors (including zero error) and random errors. (k) show an understanding of the distinction between precision and accuracy. (l) assess the uncertainty in a derived quantity by addition of actual, fractional, percentage uncertainties or by numerical substitution (a rigorous statistical treatment is not required).

Source: MOE

1. Measurement

1.1 Physical Quantities and SI Units

1.1.1 Base Quantities

Mass – kilogram(kg)
Length – metre (m)
Time – second (s)
Temperature – Kelvin (K)
Electric Current – Ampere (A)
Amount of substance – Mole (mol)

1.1.2 Derived Units

Units of other quantities derived from base units.
e.g. F=m x a (1 Newton = kg x ms^{-2})

1.1.3 Homogeneity of equations

Units of both sides of the equation must be the same.

1.1.4 Standard Form

Use standard form to deal with large/small quantities. The numerical part of a quantity is written as a single digit followed by decimal point, and as many digits after the decimal point as are justified; then multiplied by 10 to the required power.
e.g. 4.5GW = 4.5 x 10^9 W

1.1.5 SI prefixes

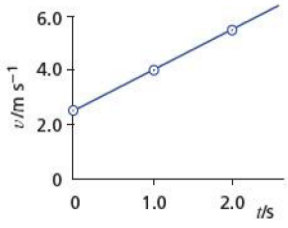
Factor	Name	Symbol	Factor	Name	Symbol
			10^{-1}	deci	d
			10^{-2}	centi	c
			10^{-3}	milli	m
			10^{-6}	micro	μ
10^{12}	tera	T	10^{-9}	nano	n
10^{9}	giga	G	10^{-12}	pico	p
10^{6}	mega	M			
10^{3}	kilo	k			

α	Α	Alpha
β	Β	Beta
γ	Γ	Gamma
δ	Δ	Delta
ε	Ε	Epsilon
ζ	Ζ	Zeta
η	Η	Eta
θ	Θ	Theta
ι	Ι	Iota
κ	Κ	Kappa
λ	Λ	Lambda
μ	Μ	Mu
ν	Ν	Nu
ξ	Ξ	Xi
ο	Ο	Omicron
π	Π	Pi
ρ	Ρ	Rho
σ or ς	Σ	Sigma
τ	Τ	Tau
υ	Υ	Upsilon
φ	Φ	Phi
χ	Χ	Chi
ψ	Ψ	Psi
ω	Ω	Omega

The Greek Alphabet

1.1.6 Graphs

t/s	v/m s^{-1}
0	2.5
1.0	4.0
2.0	5.5



Symbol for physical quantity/Units
Data in a column or along an axis
Pure numbers
If you see t/10^{2}s, in 100's

1.1.7 Order of magnitude of quantities

The power of ten to which the number is raised. Estimate to get idea of expected result.

1.2. Scalars and Vectors

1.2.1 Scalars and vectors

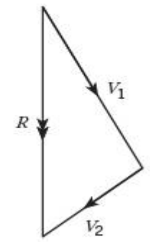
Physical quantities can be:
Scalar – magnitude only (and unit)
Vector – magnitude and direction

Quantity	Scalar	Vector
Mass	√	
Weight		√
Speed	√	
Velocity		√
Acceleration		√
Force		√
Pressure	√	
Temperature	√	

Common misunderstanding
Some students will confuse instantaneous speed and velocity with average speed for the whole journey

1.2.2 Coplanar vector addition/subtraction

V1 + V2 = R



VECTOR TRIANGLE

TIP:
REMEMBER:
(3,4,5), (5,12,13), (7,24,25)
Remember to include direction for the result

Pythagoras' theorem

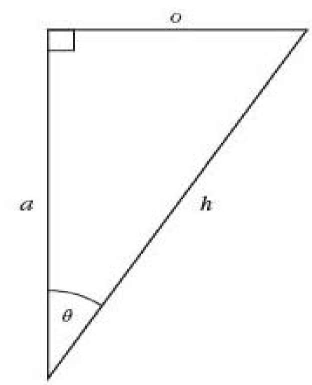


Figure 1.25

For a right-angled triangle (Figure 1.25),
$h^2 = o^2 + a^2$
Also for a right-angled triangle:
$\sin\theta = \frac{o}{h}$
$\cos\theta = \frac{a}{h}$
$\tan\theta = \frac{o}{a}$

Sine rule

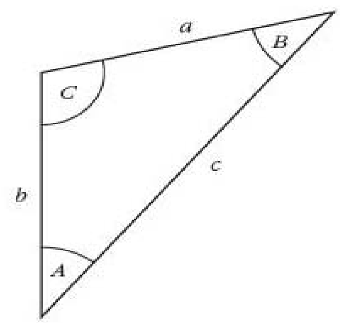


Figure 1.24

For any triangle (Figure 1.24),

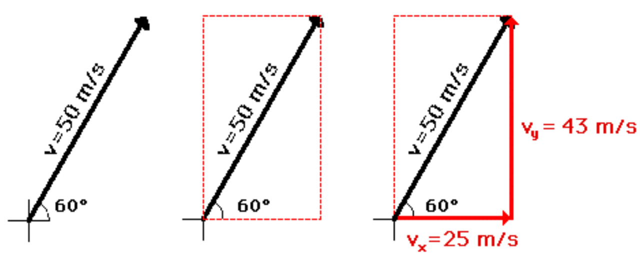
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Cosine rule

For any triangle,

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$b^2 = a^2 + c^2 - 2ac\cos B$$
$$c^2 = a^2 + b^2 - 2ab\cos C$$

1.2.3 Vector Components



Common Misunderstanding

The nose-to-tail concept of scale drawings is not easy to grasp – many students will draw a triangle and have the resultant in the wrong direction (tail-to-tail drawings).

1.3 Errors and Uncertainties

1.3.1 Systematic and random errors

Uncertainty indicates range of values within which a measurement is likely to lie

Systematic error – all readings either above or below accepted value, cannot be removed by averaging

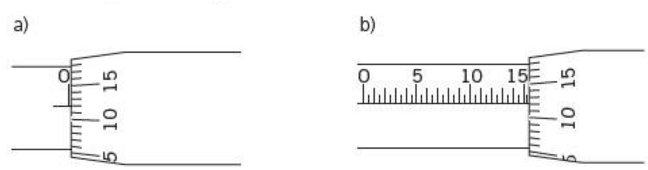
Examples include: Instrument zero error, wrongly calibrated scale, experimenter reaction time

Random error –readings scattered around accepted value. May be reduced by averaging and plotting a graph best-fit line. Examples include reading a scale, timing oscillations, quantity that varies with time, reading a scale from different angles. (If read from same angle, will result in systematic error.

Examples of uncertainty:

Instrument	uncertainty	typical reading
stopwatch with 0.1 s divisions	±0.1 s	16.2 s
thermometer with 1 °C intervals	±0.5 °C	22.5 °C
ammeter with 0.1 A divisions	±0.1 A	2.1 A

1.3.2 Micrometer Screw Gauge and parallax error



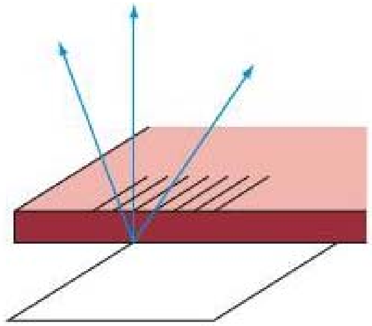
Zero error(a) = +0.12mm

Reading(b) = 15.62mm

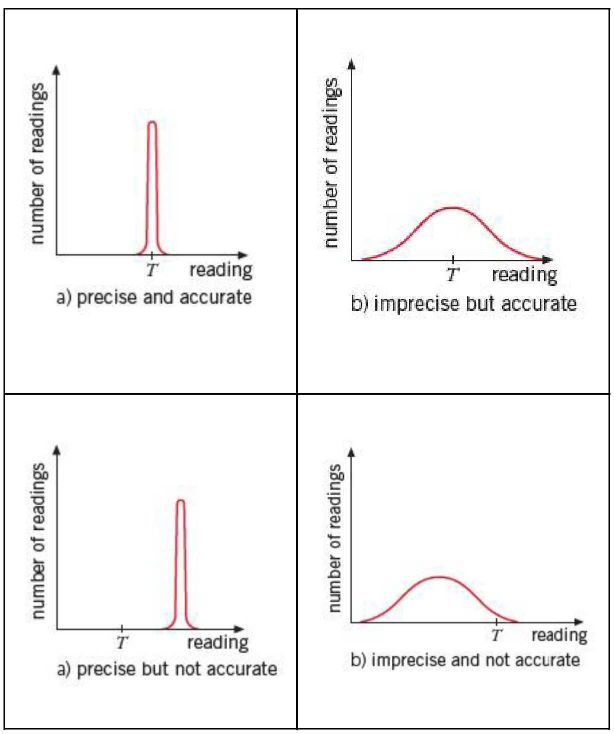
Length of object = (15.62 – 0.12)mm

= 15.50 mm

Parallax Error:



1.3.2 Precision and accuracy



Accuracy – degree measurement approaches the "true value"

Precision – size of the random error in the measurements.

1.3.3 Fractional and Percentage uncertainties

For expressions of the form:

x=y+z OR x=y-z

Overall uncertainty

$\Delta x = \Delta y + \Delta z$

For expressions of the form:

$Z=Ay^a z^b$

Overall fractional uncertainty

$\Delta x/x=a(\Delta y/y) + b(\Delta z/z)$

1.4 Other resources

http://www.physicsclassroom.com/class/vectors/Lesson-1/Relative-Velocity-and-Riverboat-Problems

This website has examples of calculating relative velocity.

http://www.launc.tased.edu.au/online/sciences/PhysSci/done/kinetics/grap_eqn/Grmotion.htm

This website has interactive resources for examining distance–time graphs.

PERSONAL NOTES:

1 What is the order of magnitude of the Young modulus for a metal such as copper?

A 10^{-11} Pa **B** 10^{-4} Pa **C** 10^{4} Pa **D** 10^{11} Pa

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2 The force F between two point charges q_1 and q_2, a distance r apart, is given by the equation

$$F = \frac{kq_1q_2}{r^2}$$

where k is a constant.

What are the SI base units of k?

A $\text{kg}\,\text{m}^3\,\text{s}^{-4}\,\text{A}^2$ **B** $\text{kg}\,\text{m}^3\,\text{s}^{-4}\,\text{A}^{-2}$ **C** $\text{kg}\,\text{m}^3\,\text{A}^2$ **D** $\text{kg}\,\text{m}^3\,\text{A}^{-2}$

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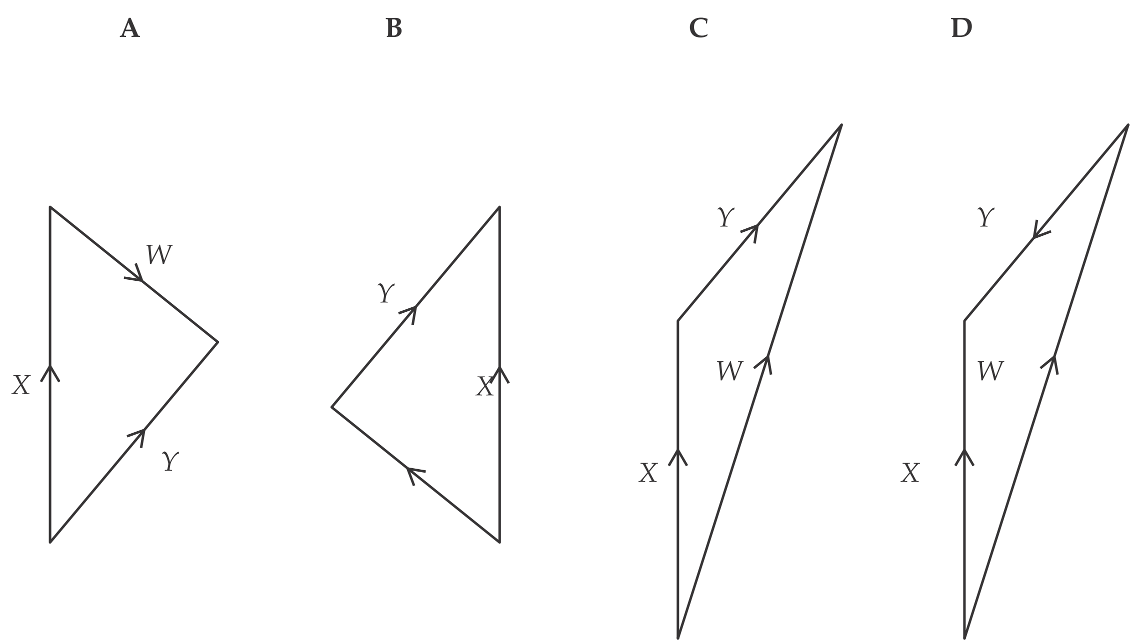
3 An aeroplane can fly at a velocity X when moving through still air. When flying in wind the aeroplane's velocity relative to the ground is Y.

Which vector diagram shows the magnitude and direction of the wind velocity W?



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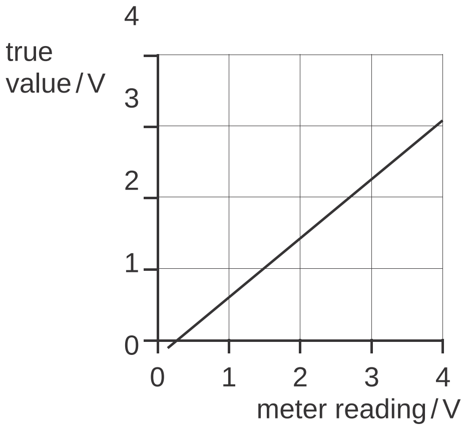
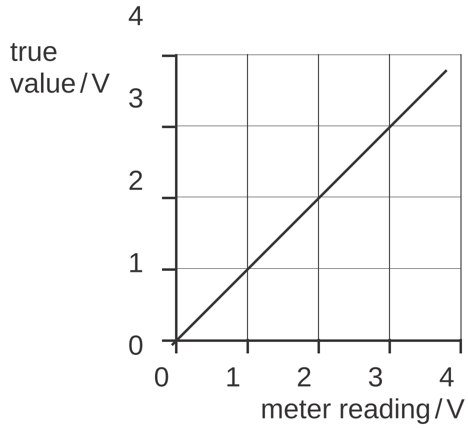
4 A voltmeter gives readings that are larger than the true values and has a systematic error that varies with voltage.

Which graph shows the calibration curve for the voltmeter?

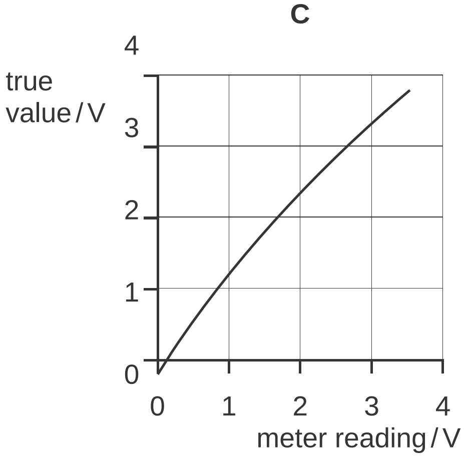
A



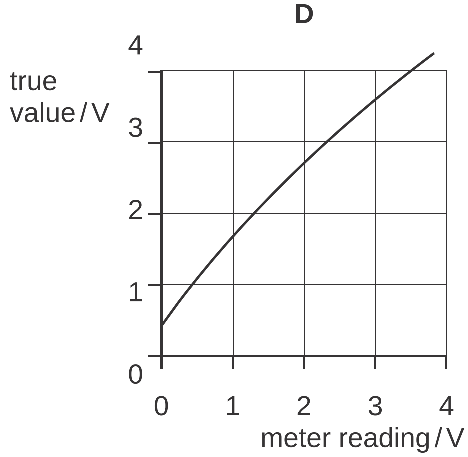
B



C



D



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5 What is the unit of weight in terms of SI base unit(s)?

A $kg\,m\,s^{-1}$ B $kg\,m\,s^{-2}$ C N D $J\,m^{-1}$

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6 At temperatures close to 0 K, the specific heat capacity c of a particular solid is given by $c = bT^3$, where T is the thermodynamic temperature and b is a constant characteristic of the solid.

The SI unit of specific heat capacity is $J\,kg^{-1}\,K^{-1}$.

What is the unit of constant b, expressed in SI base units?

A $m^2\,s^{-2}\,K^{-3}$

B $m^2\,s^{-2}\,K^{-4}$

C $kg\,m^2\,s^{-2}\,K^{-3}$

D $kg\,m^2\,s^{-2}\,K^{-4}$

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7 In making reasonable estimates of physical quantities, which statement is not correct?

A The frequency of sound can be of the order of GHz.

B The wavelength of light can be of the order of 600 nm.

C The Young modulus of a metal can be of the order of 10^{11} Pa.

D Beta particles are associated with one unit of negative charge.

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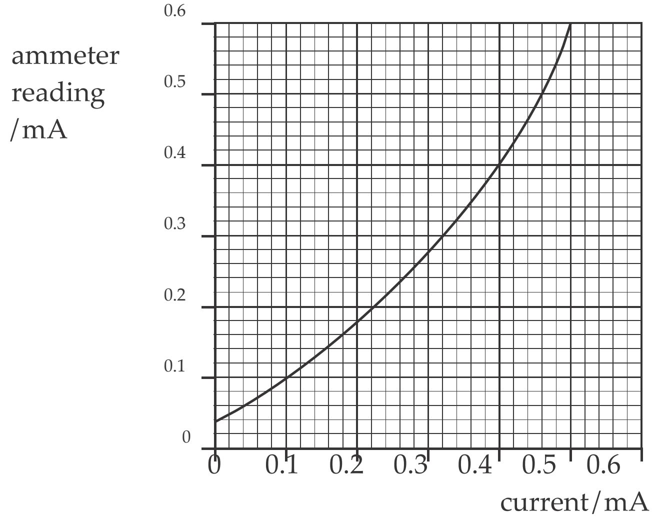
8 A calibration graph is shown for an ammeter whose scale is inaccurate.



Two readings taken on the meter at different times during an experiment are 0.13 mA and 0.47 mA.

By how much did the current really increase between taking the two readings?

A 0.30 mA B 0.35 mA C 0.40 mA D 0.44 m

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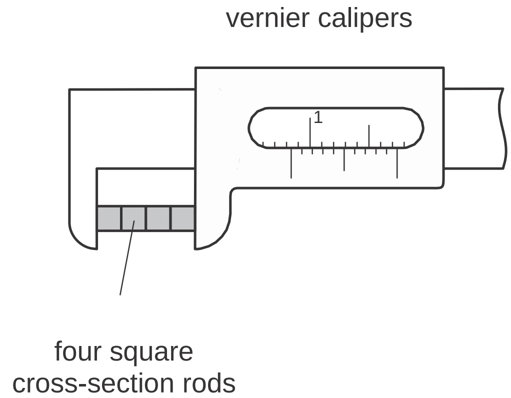
9 Four identical rods have a square cross-section. The rods are placed side by side and their total width is measured with vernier calipers, as shown.



The measurement is (8.4 ± 0.1) mm and the zero reading on the calipers is (0.0 ± 0.1) mm.
What is the width of one rod?

A (2.10 ± 0.025) mm

B (2.10 ± 0.05) mm

C (2.1 ± 0.1) mm

D (2.1 ± 0.2) mm

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10
H16

Which of the following is not a unit of energy?

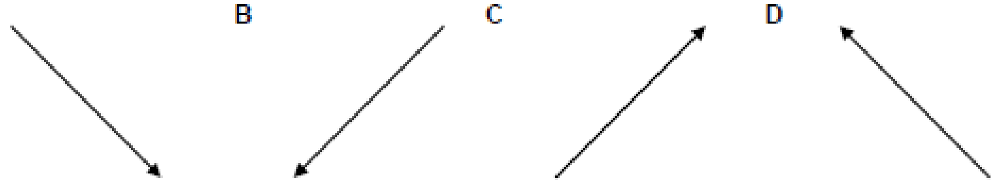
A W s   B N m   C kW h   D N s^{-1}

The initial velocity of an object is shown by the vector ***u***. The final velocity of the object is shown by the vector ***v***.

11
H16

Which arrow shows the change in velocity of the object?

A   B   C   D



12
N16

The volume of liquid flowing per second is called the volume flowrate Q and has the unit m^3 s^{-1}. The flowrate through a hypodermic needle during an injection can be estimated with the following equation:

$$Q = \frac{\pi R^n (P_2 - P_1)}{8\eta L}$$

The length and radius of the needle are L and R, respectively. The pressure at opposite ends of the needle are P_2 and P_1. The viscosity of the liquid is given by η which has the unit kg m^{-1} s^{-1}. The value of n is

A 2   B 3   C 4   D 8

STRUCTURED QUESTIONS

1 **(a)** Define *density*.

....

....[1]

(b) The mass m of a metal sphere is given by the expression

$$m = \frac{\pi d^3 \rho}{6}$$

where ρ is the density of the metal and d is the diameter of the sphere.

Data for the density and the mass are given in Fig. 1.1.

quantity	value	uncertainty
ρ m	8100 kg m^{-3} 7.5 kg	± 5% ± 4%

Fig. 1.1

(i) Calculate the diameter d.

d = ... m [1]

(ii) Use your answer in **(i)** and the data in Fig. 1.1 to determine the value of d, with its absolute uncertainty, to an appropriate number of significant figures.

d = ... ± .... m [3]

[Total: 5]

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2 **(a)** State two SI base quantities other than mass, length and time.

1. ....

2. ....
[2]

(b) A beam is clamped at one end and an object X is attached to the other end of the beam, as shown in Fig. 1.1.

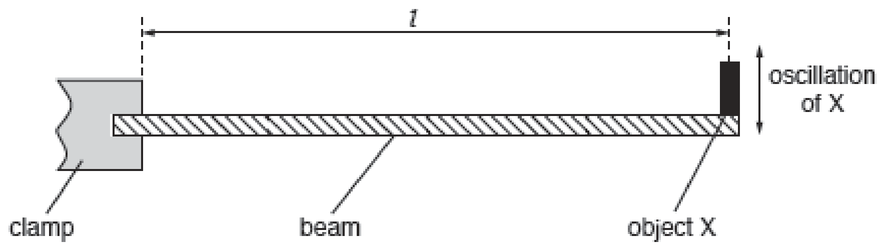

Fig. 1.1

The object X is made to oscillate vertically.

The time period T of the oscillations is given by

$$T = K\sqrt{\frac{Ml^3}{E}}$$

where M is the mass of X,
l is the length between the clamp and X,
E is the Young modulus of the material of the beam
and K is a constant.

(i) **1.** Show that the SI base units of the Young modulus are kg m^{-1} s^{-2}.

[1]

2. Determine the SI base units of K.

SI base units of K .... [2]

(ii) Data in SI units for the oscillations of X are shown in Fig. 1.2.

quantity	value	uncertainty
T	0.45	± 2.0%
l	0.892	± 0.2%
M	0.2068	± 0.1%
K	1.48×10^5	± 1.5%

Fig. 1.2

Calculate E and its actual uncertainty.

E = ... ± .... $kg\,m^{-1}\,s^{-2}$ [4]

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3 **(a)** Mass, length and time are SI base quantities.
State two other base quantities.

1. ....

2. ....
[2]

(b) A mass m is placed on the end of a spring that is hanging vertically, as shown in Fig. 1.1.

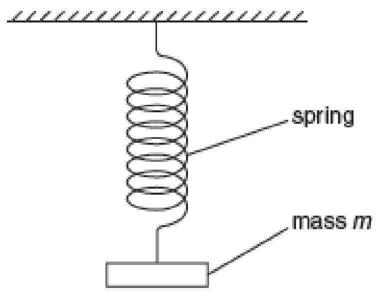

Fig. 1.1

The mass is made to oscillate vertically. The time period of the oscillations of the mass is T.

The period T is given by

$$T = C\sqrt{\frac{m}{k}}$$

where C is a constant and k is the spring constant.

Show that C has no units.

[3]

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4

(a) Define *pressure*.

.... [1]

(b) A cylinder is placed on a horizontal surface, as shown in Fig. 2.1.

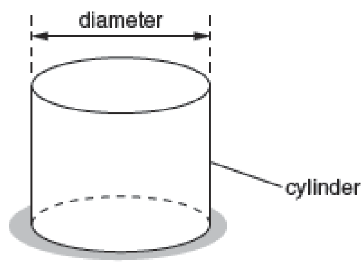

Fig. 2.1

The following measurements were made on the cylinder:

mass = 5.09 ± 0.01 kg
diameter = 9.4 ± 0.1 cm.

(i) Calculate the pressure produced by the cylinder on the surface.

pressure = ... Pa [3]

(ii) Calculate the actual uncertainty in the pressure.

actual uncertainty = ... Pa [3]

(iii) State the pressure, with its actual uncertainty.

pressure = ... ± .... Pa [1]

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5
N14

(a) The travelling microscope may be read to ± 0.1 mm. Discuss whether it would be possible to use this apparatus to detect a variation of 1% in the diameter of the tube between two points, 25 mm apart.

....

....

.... [2]

(b) To reduce random error, we can take the average of several readings of a quantity. Explain why taking average of a few readings can reduce random error.

....

....

.... [2]

(c) 'Velocity is a vector quantity, and kinetic energy of a body is equal to half its mass times the square of its velocity. Hence, kinetic energy must also be a vector.' Comment on the correctness of this statement.

....

....

.... [2]

(a) State the conditions necessary for the equations of motion, $v = u + at$ and $v^2 = u^2 + 2as$, to be applicable.

....

.... [2]

(b) An elevator starts at rest on the ninth floor. At t = 0, a passenger pushes a button to go to another floor. **Fig 2.1** shows the acceleration, a_y, of the elevator as a function of time.

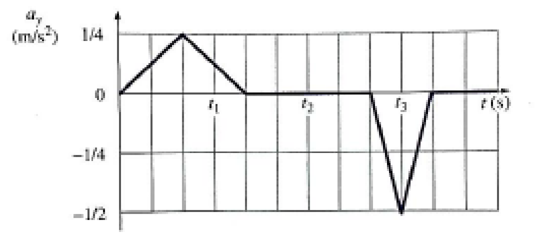

Fig 2.1